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SPHINXCAM-PRO E: COMPUTER-AIDED MODELING & MANUFACTURING OF SPHERICAL MECHANISMS VIA THE WEB

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ABSTRACT

In this paper we present a web-based computer-aided design modeling and manufacturing methodology for spherical mechanisms. Our purpose is to facilitate the analysis, dynamic simulation, and manufacture of one degree of freedom spherical four-bar mechanisms. First, a brief review of some of the current computer-aided design (CAD) software for spherical fourbar mechanisms, e.g. SPHINX, SPHINXPC, ISIS, and OSIRIS is presented. These software packages provide the three-dimensional visualization and computational capabilities necessary to design spherical four-bar mechanisms. However, to date no readily available and effective tools exist to aid in the modeling and manufacture of spherical mechanisms. Next, the kinematics of spherical fourbar mechanisms are reviewed as they pertain to their geometric modeling and manufacture. Finally, we present our web-based implementation of a computer-aided modeling, simulation, and manufacturing methodology for spherical four-bar mechanisms called SPHINXCAM-Pro|E. SPHINXCAM-Pro|E, when used with the CAD tools mentioned above, facilitates the design, dynamic simulation, prototyping, and manufacture of spherical four-bar mechanisms.

INTRODUCTION

Traditional one degree of freedom four-bar linkages are capable of generating only planar movements. Spherical four-bar mechanisms produce motion that is constrained to the surface of a sphere while still only having one degree of freedom, see Fig. 10. This complex motion is desirable since the mechanism can be designed to move a body through many orientations while still being driven by a single motor. It is often stated that spherical mechanisms are challenging to design, visualize, prototype and manufacture. The synthesis, analysis, design, and visualization problems have been solved to some extent by the CAD programs such as SPHINX (Larochelle et al 1993), SPHINXPC (Ruth and McCarthy 1997), Isis (Larochelle et al 1998) (Furlong et al 1999), and Osiris (Tse, 2000, Tse and Larochelle 1999 & 2000). Even though these packages facilitate the synthesis and analysis of spherical mechanisms they do not assist the important subsequent stages of prototyping, testing, and fabrication (Mavroidis et al 2001). As stated by (Laliberté et al 2001) "... the design and fabrication of a prototype using traditional techniques is rather long, tedious, and costly". The manufacturing methodology proposed here facilitates the prototyping and manufacturing of spherical mechanisms. The challenges of manufacturing spherical four-bar mechanisms that the methodology address are: precise link arc lengths, precise radial link placement, accurate orientation of axes, compactness of the mechanism, and a complete and consistent solid geometry model.

The methodology presented here yields accurate axes location for the links and lays out the mechanism with compact circular arcs. Precise axis placement is vital to the manufacturing of spherical mechanisms since inaccuracies may result in the link

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not rotating at the proper radius, which may in turn lead to increased friction and/or binding of the mechanism. Circular arcs are used because they yield a compact mechanism and when machined permit the links to be spaced closely together which reduces internal loading and conserves material.

It is often stated that once a mechanism has been designed that it is desirable to construct a fully functioning prototype (Norton 2004 and Alam et al 1999). The web-based methodology presented here can be used to: (1) layout the final design of a mechanism, (2) the layout may be used to quickly manufacture a fully functioning physical prototype, or (3) the assembly file maybe used to generate a virtual prototype of the device. A webbased tool for planar mechanisms was presented in Cheng 2004 and a web-based software for spatial cam design was discussed in Yin 2002. SPHINX CAM-Pro E is an implementation of this methodology that utilizes ProENGINEER Wildfire 2.0 and ASP.NET to provide a free online tool for the design of one degree of freedom spherical part orienting devices. SPHINXCAM-Pro E provides the designer with industry standard file formats (dxf, step, iges, stl) that can be loaded into computer numerically controlled (CNC) machining centers for manufacturing. Moreover, it also provides designers with a three-dimensional rendered image of their assembled design (tiff, jpg). Using these automated and accurate tools facilitates the manufacturing and simulation of spherical mechanisms with tight machining tolerances on the critical dimensions (e.g. link arc length, axis placement and orientation).

SPHERICAL FOUR-BAR MECHANISMS

Traditional planar four-bar mechanisms are a one degree of freedom closed kinematic chain connected by four revolute (R) joints. The joint axes are all parallel and the mechanism is modelled in a single plane. A spherical four-bar mechanism is also a one degree of freedom closed kinematic chain connected by four revolute joints. However, for spherical mechanisms the joint axes all intersect at a common point. This point is the center of the sphere that the mechanism moves about (Chiang 1992). In both cases the mechanisms can be designed in two dimensions: on the plane or on the sphere, see Fig. 1 (a). However, when you need to manufacture and assemble the links of the planar four-bar they can no longer be coplanar. Instead they must lie in parallel planes, see Fig. 1 (b). Similarly, for spherical four-bar mechanisms the links must be manufactured and assembled to operate in concentric spheres. Manufacturing links to operate in concentric spheres with accurate link lengths and in a compact mechanism is facilitated by the web-based CAM methodology presented here.

In spherical kinematics a link is characterized by the great circle arc subtended by its two joint axes. The two great circles associated with two adjacent links intersect at two points on the sphere. These two points define a line in space which is the Rjoint axis that connects the two links. Note that this line passes



Figure 1. PLANAR MECHANISM DESIGN.

through the center of the concentric spheres. Fig. 2 (a) shows the intersection of four great circles and the resulting axes of rotation and Fig. 2 (b) shows the spherical four-bar linkage axes and link nomenclature.

In spherical four-bar mechanisms the coupler link performs general spherical motion. Attaching a workpiece or tool to the coupler link usually requires additional parts. Here spherical mechanisms may require two parts, an extension or an attachment, to attach the workpiece to the coupler. The **coupler extension** is an arc length that lies in the same plane and has the same radius as the coupler, see Fig. 3. The **coupler attachment** is orthogonal to the coupler and it too has the same radius as the



a) Great Circles



Figure 3. ATTACHING THE WORKPIECE TO THE COUPLER.



Figure 2. SPHERICAL MECHANISM DESIGN.

coupler. We refer to links that operate at the same radius as being in the same layer. Hence, the coupler, coupler extension, and the coupler attachment are all in the same layer.

LINK GEOMETRY

The links of a spherical four-bar mechanism are described by their angular length along great circles, see Fig. 4 (a). Proper placement of the axes in spherical four-bar mechanisms is vital. If the locations of the axes are not accurate, then the resulting links will not rotate about the center of the concentric spheres. A link geometry that will facilitate precise link arc lengths, placement of link axes, and compactness of the mechanism is circular arcs with rectangular ends called **feet**. The feet provide a flat surface, the geometric center of which can be easily found, that facilitates the locating and machining of the axes, see Fig. 4 (b). Moreover, a jig or fixture could be used to assist in locating and machining the axes since all the feet of the mechanism are identical. By incrementing the link radii, the links can be designed to operate on different layers of the concentric spheres. This geometry solves the manufacturing and assembly problems of axis location while still keeping the mechanism compact. To complete the design process, the geometry of each link needs to be described. An arc length, radius, foot size and link width describe each link of the mechanism. The foot size and link width can be considered constant for all the links of the mechanism. The radius of each link can be determined by declaring the radius of the outer most link and then stepping in at increments of link width, cutting tool diameter and offset distance for each subsequent link. The extension and attachment of the coupler have the same radii as the coupler link and move in the same layer as the coupler. This ensures that the extension and attachment do not interfere with adjacent links on neighboring layers.



Figure 4. LINK GEOMETRY.

Coupler Extension

The coupler link has the feet necessary for axial location and manufacture, it may require that the link be extended to attach the workpiece to the mechanism. If the coupler has an extension that does not lie completely within the coupler link another foot or partial foot is added to the link. This additional foot can be used to locate and mount the coupler attachment to the coupler link. In Ketchel and Larochelle 1998 all of the five possible extension cases are addressed in detail.

Coupler Attachment

The geometry of the coupler attachment link differs from the other links. The base end that attaches to the workpiece has half of a foot. The center of the end of the half foot is the point where the workpiece is attached so that the mechanism produces the desired motion. The other end of the link has no foot. This is because it will be attached to the coupler link. The thickness of the coupler link acts as the foot of the attachment link. There are three different cases that are encountered when laying out the coupler attachment link and they are addressed in detail in Ketchel and Larochelle 1998.

SPHINXCAM Pro|E

With the use of a computer-aided design (CAD) software package such as ProENGINEER a spherical four-bar mechanism can be modelled with accurate solid geometry. *S*_{PHINX}CAM-Pro|E produces solid models that can be opened in multiple CAD packages as well as a fully assembled mechanism that can be virtually driven by motors allowing the user to visualize the motion of the mechanism without ever having to machine a part via dynamic simulation software such as ProENGINEER's *Mechanism Design* package or MSC's *Visual NASTRAN 4D*. The geometry used by *S*_{PHINX}CAM-Pro|E to create the spherical four-bar is based on a previous software package written in Autolisp entitled *S*_{PHINX}CAM that layed out the links of a spherical four-bar mechanism, see Ketchel and Larochelle 1998. *S*_{PHINX}CAM requires an Autodesk product (e.g. AutoCAD, Inventor, Mechanical Desptop) and produced a 2-D layout of the links for CNC manufacture via CAM software. In contrast, *S*_{PHINX}CAM-Pro|E is a free utility that offers a unique resource to practitioners, researchers, and students.

Graphical User Interface

The SPHINXCAM-Pro E Graphical User Interface (GUI) is the front end of a free web-based utility that can be used to create CAD models of spherical four-bar mechanisms. The GUI is a web form divided into three sections: user information, mechanism parameters (Fig. 5), and output file options (Fig. 6). The required user information and the four-bar mechanism data are stored in a log file on the server so the user may access their previously created four-bars that have been stored on the SPHINXCAM-Pro|E server. The second part of the GUI is the mechanism parameters section. Here, the user enters the dimensions and parameters for their mechanism. The GUI passes these dimensions onto ProENGINEER to create the particular spherical four-bar geometry. SPHINXCAM-Pro E displays default values for a sample crank-rocker mechanism that includes a coupler extension and attachment. If a user enters parameters that do not correspond to a four-bar that can be assembled the GUI displays an error message and highlights the parameter(s) to modify. To the right of the parameter input area are illustrations showing the physical definitions of the four-bar parameters. If a user needs more detailed information on the parameter inputs there is a hyperlink to an online help document.

The output file options allow the user to choose what file format(s) to return. *S*_{PHINX}CAM-Pro|E can return a variety of file formats including 3D assemblies (.asm, .stl, and .stp), 2D drawings (.igs and .dxf), and rendered images (.jpg and .tif). Hyperlinks to download the requested files appear in this section once the mechanism has been created. By default the ProENGINEER assembly file option is selected which returns an .asm file. In order to open the assembly file in ProENGINEER the user must download the part (.prt) files that make up the assembly. There are individual part files for the links, extensions, attachments, and axles of the four-bar. A hyperlink next to the .asm file selection downloads a compressed archive that includes the individual part files for the mechanism.

ProENGINEER

All CAD modelling for SPHINXCAM-Pro|E is been done in PTC's *ProENGINEER Wildfire 2.0*. The approach used was to create a generic parametric model of a spherical four-bar that would be updated dynamically according to dimensions input from a web user. The four-bar has been assembled as a mechanism in the software allowing the links to rotate so the user

Spherical Four-Bar	Parame	eters	Click thumbnail to enlarge.
Fixed Link Length:	80	(deg.)	
Inner Link Length:	60	(deg.)	
Outer Link Length:	40	(deg.)	
Coupler Link Length:	80	(deg.)	
Maximum Radius:	4.5	(in.)	
Link Width:	0.5	(in.)	Link Width
Tool Diameter:	0.375	(in.)	
Stock Thickness:	1.0	(in.)	
Interlink Offset:	0.125	(in.)	
Coupler Extension:	20	(deg.)	
Coupler Attachment:	30	(deg.)	

Figure 5. THE SPHINXCAM-PRO E PARAMETER INTERFACE.

File Options Download ProductView Express		
✓ Return ASM File Get Part Files ✓ Return STEP File	Download .asm File Download .stp File	Comments:
☑ Return STL File ☑ Return Image File	Download .stl File	2
○.jpg file ○.tif file		
Return 2D Drawing File .dxf file .iges file		Submit or Email Us

Figure 6. THE SPHINXCAM-PRO E OUTPUT INTERFACE.

can visualize the movement of the assembly. The first step in creating such a model was to design and draw a single spherical link, defined by the flexible parameters. Two types of axles were needed to connect the links; one long and one short. The axles not only act as a revolute joint for the links but also define the spacing between links. SPHINXCAM-Pro E also has the ability to include an extension and attachment to the coupler link. To accomplish this, multiple models of extensions and attachments were made to reflect the various geometric differences encountered for specific parameter combinations. All of the parts were modeled with relations attached to the geometry which allows for flexibility in the dimensions while keeping the design intent intact. Once all the common components had been made the fundamental four-bar was assembled using the ProMECHANISM module in ProENGINEER. This type of assembly varies from the standard form which defines rigid constraints. ProMECHA-NISM instead defines connections and joints between assembly components. These joints enable the user to drag components of the assembly and visualize the movement of the connected parts. The specific application of this method to a spherical four-bar means that a user can drag a link of the four-bar or define a rotation around a revolute joint and see the four-bar pass through its range of motion. The ProMECHANISM module also provides a way to trace the path of points, show the complete motion envelope, and analyze the kinematics of various parts of the mechanism. Users can take their downloaded assembly file into ProENGINEER and, with minimal previous knowledge, add various types of motors to drive the revolute joints and create an animation of the moving four-bar. The assembly also makes use of ProENGINEER's flexibility function. Each component of the assembly including the links, spacers, extensions, and attachments has been defined as a flexible part. This simply means that the dimensions of these parts can be modified using parameters and relations within the assembly file. For example the width of each of the links is defined as a flexible dimension so that a relation can be made equating the link width parameter of the assembly file to the actual width of each link. There are many relations made in the assembly file that link flexible dimensions of the components to parameters of the assembly file. These parameters correspond exactly to the same parameters that are input into the SPHINXCAM-Pro E GUI. By defining the parameters in ProENGINEER to be set programmatically, a text file containing all the values can be read in place of a user manually typing each one within the Pro-ENGINEER interface. This assembly file and a second one with the components laid out on a plane are the common models that adjust to meet the specific parameters provided by the user.

THE FUNDAMENTAL MECHANISM

The fundamental mechanism shown in Fig. 7 contains the links for all possible coupler extension and attachment cases. Each case handles differences in geometry for the extensions/attachments based upon their distance from the coupler. The difference in geometry for all the cases resolves any overlap problems that might occur between the extension/attachments and the coupler itself, while simultaneously keeping the links from causing interference problems.

On each end of the coupler there are four links that correspond to the four possible cases of coupler extension geometry (four on the positive side, four on the negative). In the most general case the extension is far enough away from the coupler to connect the two parts with concentric arcs, this is case 1. In case 2 the extension is closer to the coupler such that the inside edge of the extension overlaps with the coupler however the outer edge does not. In this case the outer edge is connected by an arc as in case 1 and the inner edge is connected by a line from the end of the coupler to the end of the extension. In case 3 the extension is even closer to the coupler with both the inner and outer edges overlapping with the end of the coupler. For this case the overlap is less than $\frac{1}{4}$ of the stock thickness of the mechanism, and therefore lines connect the end of the coupler to the end of the extension for both the inner and outer edges. In the final case for the extensions, the inner and outer edges of the extension overlaps with the coupler by more than $\frac{1}{4}$ of the stock thickness. On the inner edge a line normal to the end of the coupler connects to the end of the extension and the outer edge is connected as in case 3. The attachments for the coupler can be placed anywhere along the length of the coupler including any extensions. In the



Figure 7. THE FUNDAMENTAL MECHANISM.

fundamental mechanism two links on both the top and bottom sides of the coupler are attached to a datum plane that moves them to the proper location on the coupler. The two links correspond to the two cases for attachment geometry. In case 1 the attachment is far enough away from the coupler such that there is no overlap and it is connected by concentric arcs. In case 2 the attachment does overlap with the coupler and therefore it is connected by straight lines.

The four primary links of the fundamental mechanism are assembled together in Pro/Engineer using the connections tool. The fixed link is the first to be placed in the assembly and is constrained to the origin. Next the input link is assembled by placing a pin joint between one of its axes and one of the fixed using a flexible dimension for the offset. The output link is then assembled in the same fashion to the other axis of the fixed link. Finally, the coupler is added to the assembly by placing a pin joint between one if its axes and the input link and cylindrical joint between its other axis and the output link. All of the pin joints are defined with flexible dimensions to constrain the offset of the links, and to avoid over constraining the mechanism the last joint assembled must be a cylindrical joint. Once assembled in this manner, the four-bar can then be treated as a mechanism within Pro/Engineer.

Assembling the Mechanism

The piece parts of the sample spherical mechanism as layed out by $S_{\text{PHINX}}CAM$ -Pro|E are shown in Fig. 8 and the assembled mechanism is shown in Fig. 9 and Fig. 10. $S_{\text{PHINX}}CAM$ -Pro|E generated the piece part layout of Fig. 8 as well as the rendered and assembled device shown in Fig. 10.



Figure 8. THE SAMPLE MECHANISM-PART LAYOUT.



Figure 9. THE ASSEMBLED SAMPLE MECHANISM - WIRE FRAME.

Drawing the Mechanism Links

SPHINXCAM-Pro E lays out four or five links with their axial lines, depending on the data entered. If there are four links, they are laid out with one axis of each link lined up along the bottom of the screen. They are arranged, left to right, from largest to smallest radii, with their common center lying to the right. The links' arcs extend clockwise, around the common center. If there are five links, the additional link is the coupler attachment. The first four links are drawn as in the four link case and the fifth link is offset to the left and raised relative to the other links so that as it is extended clockwise, its tool paths will not interfere with the adjacent link, see Fig. 11. It has the same radius as the link that had the coupler extension. The radius of the coupler attachment link may be different from the outer-most link, which creates problems for drawing the attachment link without entering the toolpath region about the outer-most link. In order to avoid this problem the coupler attachment link is drawn offset to the left at the same spacing as the other links, and is positioned vertically according to which link has been extended and the size of the attachment angle, see Ketchel and Larochelle 1998.

The general layout of the links, including the coupler attachment case, is shown in Fig. 11. SPHINXCAM-Pro|E uses colors and



Figure 11. GENERAL LAYOUT OF LINKS.



Figure 10. THE ASSEMBLED SAMPLE MECHANISM - SOLID.

linetypes to show the axial lines of the various links. The axial lines of the regular four links have a common origin and are drawn with dashed red lines. To offset the coupler extension axial line, since it also intersects the common center, *S*_{PHINX}CAM-Pro|E uses a different linetype that is blue. If there is a coupler attachment its axial lines are drawn with another linetype that is green.

CASE STUDY

SPHINXCAM-Pro|E was used to layout the links for the Infinity Fan, see Larochelle et al 2001. The data for constructing the fan's spherical mechanism was generated with *S*PHINX, see Tbl. 1, and combined with the desired stock thickness, link width, offset distance, and cutting tool diameter. *S*PHINXCAM-Pro|E produced the drawings, see Fig. 12, that were then exported into a CAM package that generated the CNC code to manufacture the links of the mechanism. The completed Infinity Fan is shown in Fig. 13.

CONCLUSIONS

We presented a web-based computer-aided modeling and manufacturing software tool for spherical mechanisms entitled *S*_{PHINX}CAM-Pro|E. *S*_{PHINX}CAM-Pro|E uses computer-aided drafting and manufacturing to address many of the challenges encountered when building spherical mechanisms. *S*_{PHINX}CAM-Pro|E provides a free web-based interface to software that generates solid geometric models and an assembly of the mechanism using ProENGINEER Wildfire 2.0. The links are designed to facilitate accurate axis placement which is critical to spherical mechanisms. Moreover, *S*_{PHINX}CAM-Pro|E creates compact links which reduces internal loading and conserves raw materials. Finally, the solid geometry assembly and can be used for virtual prototyping and dynamic simulation. *S*_{PHINX}CAM-Pro|E is available at

Iable 1. Infinity Fan Mechanism Data					
SPHINX Data					
Link/Prompt	Angle/Value				
Largest Radii Link	88.284 (deg)				
Second Largest Radii Link	53.806 (deg)				
Second Smallest Radii Link	47.350 (deg)				
Smallest Radii Link	87.894 (deg)				
Link to Extend	1				
Link Extension	-86.350 (deg)				
Link Attachment	-46.690 (deg)				
Other Data Used					
Prompt	Value Entered				
System of Units	1 (U.S. Customary)				
Stock Thickness	1 (in)				
Cutting Tool Diameter	0.375 (in)				
Off-set Distance	0.125 (in)				
Link Width	0.500 (in)				
Maximum Radius	4.250 (in)				
SPHINXCAM-Pro E Output					
Spacer sizes, two of each needed:					
0.5 and 1.5 (in)					

http://my.fit.edu/~pierrel/software.html.

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REFERENCES

- Larochelle, P. M., Dooley, A. P., Murray, A. P., and Mc-Carthy, J. M., 1993. "Sphinx: Software for synthesizing spherical 4r mechanisms". *NSF Design and Manufacturing Systems Conference*, **1**, January, pp. 607–611.
- [2] Ruth, D. A., and McCarthy, J. M., 1997. "Sphinxpc: An implementation of four position synthesis for planar



Figure 12. SPHINXCAM-Pro E INFINITY FAN LINK LAYOUT.



Figure 13. INFINITY FAN-TOP VIEW.

and spherical linkages". ASME Mechanisms Conference, September.

- [3] Larochelle, P. M., Vance, J. M., and McCarthy, J. M., 1998. "Creating a virtual reality environment for spherical mechanism design". *NSF Design and Manufacturing Grantees Conference*, January, pp. 83–84.
- [4] Furlong, T. J., Vance, J. M., and Larochelle, P. M., 1999.
 "Spherical mechanism synthesis in virtual reality". *ASME Journal of Mechanical Design*, **121**(4), December, pp. 515–520.
- [5] Tse, D., 2000. *Computer-Aided Synthesis of Mechanical Systems for Spatial Motion*. Dissertation, Florida Institute of Technology.
- [6] Tse, D., and Larochelle, P., 1999. "Osiris: A new generation spherical and spatial mechanism cad program". *Pro-*

ceedings of the 1999 Florida Conference on Recent Advancements in Robotics, April 29-30.

- [7] Tse, D., and Larochelle, P., 2000. "Approximating spatial locations with spherical orientations for spherical mechanism design". ASME Journal of Mechanical Design, December, pp. 457–463.
- [8] Mavroidis, C., DeLaurentis, K., Won, J., and Alam, M., 2001. "Fabrication of non-assembly mechanisms and robotic systems using rapid prototyping". *ASME Journal* of Mechanical Design, **123**, December, pp. 516–524.
- [9] Laliberté, T., Gosselin, C., and Côté, G., 2001. "Practical prototyping". *IEEE Robotics & Automation Magazine*, September, pp. 43–52.
- [10] Norton, R., 2004. *Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines*, third ed. McGraw-Hill.
- [11] Alam, M., Mavroidis, C., Langrana, N., and Bidaud, P., 1999. "Mechanism design using rapid prototyping". In TENTH World Congress on the Theory of Machines and Mechanisms, Vol. 3, IFTOMM, pp. 930–938.
- [12] Cheng, H., and Trang, D., 2006. "Web-based mechanism design and analysis". *Journal of Computing and Information Science in Engineering*, 6(1), March, pp. 84–90.
- [13] Yin, G., and Tian, G., 2002. "A web-based remote cooperative design for spatial cam mechanisms". *International Journal of Advanced Manufacturing Technology*, 8(2), September, pp. 557–563.
- [14] Chiang, C. H., 1992. "Spherical kinematics in contrast to planar kinematics". *Mech. Mach. Theory*, 27(3), pp. 243– 250.
- [15] Ketchel, J., and Larochelle, P., 1998. "Sphinxcam: Computer-aided manufacturing for spherical mechansims". ASME Mechanisms Conference. Paper # MECH-5886.
- [16] Larochelle, P., Dees, S., and Ketchel, J., 2001. *Infinity Fan.* US Patent #6213715.